

Unusual Constraints in the Quantum Statistical Mechanics of Josephson Junction Systems

Taner Edis¹

Received May 15, 1992; final June 18, 1992

In order to apply quantum statistical mechanics to systems composed of Josephson junctions, the unconventional constraint of fixed "macroscopic wave function" magnitudes on either side of a junction must be accommodated. In order to use this information, the density matrix formalism must be extended to deal directly with probability distributions over general quantum states. As a result, in thermal equilibrium, the explicit temperature dependence becomes modified from the trivial $1/kT$ factors.

KEY WORDS: Maximum-entropy formalism; Josephson junctions; quantum and information entropies.

1. INTRODUCTION

The behavior of systems composed of superconducting areas connected to each other via Josephson junction-type weak links is of present interest. Granular materials are common; indeed, high- T_c ceramics are also in general made up of superconducting grains separated by such links.⁽¹⁾ There have also been investigations of less disorderly Josephson junction arrays,⁽²⁾ motivated partially by possible device applications of such systems.⁽³⁾ What is common to these studies is a collection of junctions, typically in the presence of an external applied magnetic field.

A first step in approaching such assemblages would be to determine equilibrium behavior, by their statistical mechanics, though eventually the transport properties are more interesting. While this has been done in a conventional manner,⁽⁴⁾ the macroscopic quantum nature of the Josephson junction phenomenon allows development of a quantum statistical description that is subtly different in its results.

¹ Department of Physics and Astronomy, Johns Hopkins University, Baltimore, Maryland 21218.

2. STATISTICAL QUESTIONS ABOUT JUNCTIONS

A detailed treatment of a junction between two superconductors separated by a thin insulating layer involves tunneling of electron pairs across the barrier. But with the pair creation and annihilation operators "referring to macroscopically occupied states,"⁽⁵⁾ a good approximation is available with macroscopic wave functions. $\psi = n_s^{1/2} e^{i\theta}$ has the superconducting pair density n_s as its magnitude, and is uniform in space within type I superconductors. So a Josephson junction can be approximated by a coupled quantum two-state system,⁽⁶⁾ with an overall wave function having the macroscopic wave functions on either side as components:

$$\psi = \begin{pmatrix} n_1^{1/2} e^{i\theta_1} \\ n_2^{1/2} e^{i\theta_2} \end{pmatrix}, \quad H = \begin{pmatrix} \varepsilon & \kappa \\ \kappa & -\varepsilon \end{pmatrix} \quad (1)$$

The coupling strength is κ , and $\varepsilon = qV/2$, with V the applied voltage, is half the energy difference between pairs on either side. Also, in the following discussion, the normalization of ψ will be taken to be unity, $n_1 + n_2 = 1$, in order to keep the usual normalization of general quantum mechanics. In all expressions, energies and energy densities may be connected by insertion of a proper overall density factor. The Schrödinger equation $i \partial_t \psi = H \psi$ (setting $\hbar = 1$) can now be solved. Following Feynman,⁽⁶⁾ a set of four differential equations is obtained (separating real and imaginary parts),

$$\begin{aligned} \dot{\theta}_1 &= (n_2/n_1)^{1/2} \kappa \cos \phi - \varepsilon, & \dot{\theta}_2 &= -(n_1/n_2)^{1/2} \kappa \cos \phi + \varepsilon \\ (\dot{n}_1)^{1/2} &= (n_1 n_2)^{1/2} \kappa \sin \phi, & (\dot{n}_2)^{1/2} &= -(n_1 n_2)^{1/2} \kappa \sin \phi \end{aligned} \quad (2)$$

which lead to the familiar Josephson junction current equation $J = J_c \sin \phi$, J_c being a critical current, and with $\phi = \theta_1 - \theta_2$ the superconducting phase difference across the junction. The argument is that the time dependence of the pair density represents a *tendency* to change, the constant current flow in a steady state continually restoring the balance of pairs on either side. The energy is also easily found to be

$$\langle \psi | H | \psi \rangle = (n_1 - n_2) \varepsilon + 2\kappa (n_1 n_2)^{1/2} \cos \phi \quad (3)$$

giving rise to the familiar junction energy expression, $E \sim \cos \phi$.

The densities n_1 and n_2 are determined by the temperature and material characteristics. Thus, ϕ is the only true variable in the two-state system described, as the possible states ψ are restricted to those with fixed magnitudes n_1 and n_2 for its two components in the basis of (1). In particular, it is clear that in general the eigenstates of H are not physically realizable, and only a subset of the state space spanned by them satisfies

the constraint. Thus the question arises of how a form of quantum statistical mechanics could be applied to the junction, assuming that the macroscopic wave function picture remains valid.

For the statistical mechanics of Josephson junction arrays, the usual procedure is to take the classical system with an energy expression such as (3) for each junction, giving rise to variants of frustrated XY models.^(2,4) With the Hamiltonian

$$H = \sum_{\langle ij \rangle} J_{ij} \cos(\theta_i - \theta_j - A_{ij}) \quad (4)$$

(A_{ij} arising from the presence of a magnetic field, to ensure gauge invariance), a partition function can be written down⁽⁴⁾

$$Z = \int \prod_i d\theta_i e^{-H(\theta)/kT} \quad (5)$$

The phases are treated as classical variables in a canonical ensemble.

Any quantum accounting for the statistics will have to incorporate the rather peculiar constraint of a restriction of states to those with fixed pair densities on either side. These allowed states are not stationary under the dynamics determined by the H of (1), but the steady-state current flow argument applies. When $\varepsilon=0$, it is correct to say that the state does not evolve into another, by (2). Thus, when no external voltage is applied, any existing phase difference does not change, and the probability distribution for this difference will have to reflect the extra available information, on the fixed densities.

A *physically* constrained system may be described by instantaneous results extended over all times, obtained by a constraint on allowed states instead of an explicit change in the Hamiltonian. A simpler example, with a classical gas in a box, might help to clarify matters. The statistical mechanics of a system with the same gas in a box half the size is related to the full-size problem. One can either change the physical constraints and solve the new problem directly, or restrict the allowed state space to those with molecules only in one half of the box. In this case, the extra information can be utilized to obtain an “instantaneous” probability distribution for the gas. Of course, in the full-size box this will not be a stationary distribution, as molecules will rapidly diffuse into the other half. But if this distribution is taken to apply at all times, one has the solution to the half-size problem. For Josephson junctions, the results from (1) are used, ignoring the details of how physically a steady state is achieved for states nonstationary under the original Hamiltonian.

3. STATE PROBABILITIES

In the usual case, description of the statistical mechanics of a quantum system is done with the density matrix $\rho = \sum_i P_i |\psi_i\rangle\langle\psi_i|$, where the P_i denote probabilities of being in states $|\psi_i\rangle$, which need not be mutually orthogonal. This operator ρ contains all the information necessary to compute any operator expectation value, defined as the double average

$$\langle\langle O \rangle\rangle \equiv \sum_i P_i \langle\psi_i| O |\psi_i\rangle = \text{Tr}(\rho O) \quad (6)$$

Thus, the initial task to face in a problem is setting up the density matrix, which is essentially a question of assigning probabilities that accurately reflect the information to be used about the system. This can be done by use of the “maximum-entropy” or “maxent” formalism, where we choose ρ such that it is the *least informative* among those satisfying our constraints. Shannon’s information entropy⁽⁷⁾ is a unique measure of missing information that we can then require to be maximized.

With expectation value constraints, we easily obtain the desired maxent ρ ’s, using Lagrange multipliers and the entropy expression $\sigma = -\text{Tr} \rho \log \rho$, maximizing

$$\sigma + \lambda_0(1 - \langle\langle 1 \rangle\rangle) + \sum \lambda_i(\tilde{O}_i - \langle\langle O_i \rangle\rangle) \quad (7)$$

with the \tilde{O}_i denoting values that the expectations of the operators O_i are forced to be. The λ_0 condition is to normalize the probabilities, i.e., to have $\text{Tr} \rho = 1$. The solution is the familiar

$$\rho = \frac{1}{Z} \exp\left(-\sum \lambda_i O_i\right) \quad (8)$$

$$Z = \text{Tr} \exp\left(-\sum \lambda_i O_i\right) = \exp(\lambda_0) \quad (9)$$

becoming the canonical distribution when the expectation value of the Hamiltonian operator is constrained.

Having summarized the use of the density matrix in mean value constrained statistical mechanical problems, we must also consider the origin of the expression $\sigma = -\text{Tr} \rho \log \rho$ for the quantum entropy. For a classical system that has discrete states, the information entropy expression is straightforward, $-\sum_i p_i \log p_i$, with the p_i the probabilities of each state. Various expressions have been proposed to extend this to the quantum case. Jaynes⁽⁸⁾ discusses some, and decides on σ as being the only satisfactory one.

The straightforward extension $S = -\sum_i P_i \log P_i$, with P_i the original probabilities of (in general nonorthogonal) states, is rejected in favor of σ , "because the P_i are not in general the probabilities of mutually exclusive events."⁽⁸⁾ If the state is known to be $|\psi\rangle$, the probability of measuring it to be in state $|\phi\rangle$ is $|\langle\phi|\psi\rangle|^2$. However, keeping in mind the conceptual distinction between "being in" and "observed in," there is no reason that working with the state probabilities $P(\psi) d\psi$ should be invalid.

If the information we are given, and that we hope to extract, is purely in the form of expectation values, the density matrix obtained by maximizing σ will provide, most economically, all that is needed. In almost all cases of physical interest, expectations are all we have, or, certain symmetry conditions exist that can be used to restrict states that provide a diagonal basis for ρ , such as fermionic or bosonic requirements. These, too, are easily accommodated within the conventional framework.

But the statistical mechanics can be formulated at the level of the most basic available description, that of state probabilities. That this more general formulation is physically equivalent to working with the density matrix entropy expression σ , when applicable, will be demonstrated in the following sections. Such an approach, which corresponds to the classical "summing over states" procedure, is necessary for the Josephson junction problem. A restriction on allowed states is easy to implement in such a picture. But this is not a constraint expressible in the form of projection operator expectations, so trying to obtain a density matrix directly via maximizing σ is impossible.

4. MAXIMUM ENTROPY ON $P(\psi)$

An expression for information entropy as a functional of $P(\psi)$ is needed; this will be of the form

$$S[P(\psi)] = - \int d\psi P(\psi) \log \frac{P(\psi)}{M(\psi)} \quad (10)$$

where $M(\psi)$ is a prior probability distribution, necessary for continuous distribution entropies, to make the expression invariant under changes of variable.^{(9),2} In an orthonormal basis, $|\psi\rangle = \sum c_i |i\rangle$, the ψ in (10) representing a set of such coefficients c_i . Thus, with the normalization constraint,

$$\int d\psi \equiv \int \prod_i dc_i dc_i^* \delta \left(1 - \sum_j |c_j|^2 \right) \quad (11)$$

² Invariance under a transformation group is taken to be a rigorous way to define an uninformative prior.

The prior $M(\psi)$ is analogous to the Bayesian “uninformative prior,” the distribution in the case that no information at all is available; thus the expression (10) is an entropy relative to an uninformed distribution. In most cases of physical interest, a requirement of invariance under a transformation group sufficiently describes a condition of indifference that fixes this prior.⁽⁹⁾

$M(\psi)$ can be computed by realizing that in the completely uninformed case, there can be no difference in what orthogonal basis is selected to give the coefficients c_i ; i.e., $M(\psi)$ is invariant under changes of variable brought about by the action of the unitary group $U(n)$ on state vectors, n being the dimensionality of the state space. The normalization condition is that $\sum |c_j|^2 = 1$ has state vectors lying in an “ n -sphere” in \mathcal{C}^n , where one state is transformed to another by the action of members of $U(n)$. Therefore, $M(\psi)$ will have to be a constant on this surface, corresponding to the intuitive notion of the probability being equal for all states, there being no reason to discriminate between them. So (10) becomes

$$S[\psi] = \log M - \int d\psi P(\psi) \log P(\psi) \quad (12)$$

The constant $\log M$ can be ignored, as we are interested in the maxima of S .

It has to be demonstrated, for consistency, that when S is maximized subject to expectation value constraints, the result is physically equivalent to what would be obtained through a conventional σ maximization. For this, it will suffice to show that the density matrix obtained from a maximum- S probability distribution is equal to the one in (8).

Maximizing subject to constraints

$$\begin{aligned} & - \int d\psi P(\psi) \log P(\psi) + \gamma_0 \left[1 - \int d\psi P(\psi) \right] \\ & + \sum_i \gamma_i \left[\tilde{O}_i - \int d\psi P(\psi) \langle \psi | O_i | \psi \rangle \right] \end{aligned} \quad (13)$$

gives

$$\begin{aligned} P(\psi) d\psi &= \frac{1}{Q} \exp \left(- \sum \gamma_i \langle \psi | O_i | \psi \rangle \right) d\psi \\ Q &= \int d\psi \exp \left(- \sum \gamma_i \langle \psi | O_i | \psi \rangle \right) \end{aligned} \quad (14)$$

Q is the analogue of the partition function, in that

$$\langle\langle O_i \rangle\rangle = - \frac{\partial \log Q}{\partial \gamma_i} \quad (15)$$

By using the Fourier integral representation for the delta function in (11), and then doing the contour integral after the Gaussian integrals, we can obtain Q in closed form:

$$Q = \sum_i \frac{e^{-\Gamma_i}}{\prod_{j \neq i} (\Gamma_j - \Gamma_i)} \quad (16)$$

with a multiplicative constant ignored. The Γ_i are the (nondegenerate) eigenvalues of the operator $\Gamma = \sum \gamma_i O_i$. In general the density matrix elements are obtainable by

$$\rho_{ij} = - \frac{\partial \log Q}{\partial \Gamma_{ij}} \quad (17)$$

The expressions obtained by application of (17) are not such that equivalence of the density matrices obtained through maximization of σ and S , $\rho^{(\sigma)}$ and $\rho^{(S)}$, is immediately obvious beyond the fact that they are diagonal in the same basis. However, this can be demonstrated. In the following, only one operator is taken to be fixed in expectation, the Hamiltonian, with eigenvalues ε_i . This is for the sake of clarity and concreteness only, and can be readily generalized.

The maximum S density matrix and the canonical density matrix each have a single parameter, determined by the energy expectation constraint:

$$\rho_i^{(S)}(\gamma \varepsilon_1, \gamma \varepsilon_2, \dots) = - \frac{\partial \log Q}{\partial (\gamma \varepsilon_i)} \quad (18)$$

$$\rho_i^{(\sigma)}(\beta \varepsilon_1, \beta \varepsilon_2, \dots) = - \frac{\partial \log Z}{\partial (\beta \varepsilon_i)}$$

$$- \frac{\partial \log Q}{\partial \gamma} = - \frac{\partial \log Z}{\partial \beta} = \tilde{E} \quad (19)$$

The β in $\rho^{(\sigma)}$ is the usual inverse temperature. If $\rho_i^{(\sigma)}(\beta \varepsilon) = \rho_i^{(S)}(\gamma \varepsilon)$ is to hold for all i , each of the functions $\gamma(\beta)$ implicitly defined by these relations must be identical. This will be ensured by

$$\frac{\partial \rho_i^{(\sigma)}}{\partial \gamma} \frac{\partial \rho_j^{(S)}}{\partial \beta} = \frac{\partial \rho_j^{(\sigma)}}{\partial \gamma} \frac{\partial \rho_i^{(S)}}{\partial \beta}, \quad \forall i, j \quad (20)$$

as the above implies $\partial \gamma / \partial \beta$ is the same for each i , and neither density matrix contains terms independent of their parameters, γ or β . Using (18) for the ρ 's, interchanging the order of differentiation, and then using (19) establishes the desired result.

While a γ can be found to give any density matrix parametrized by a particular inverse temperature β , in general the function $\gamma(\beta)$ cannot be obtained analytically.

State-level statistical descriptions are thus equivalent to the density matrix form when expectation value constraints are present. The parameter γ is physically the functional equivalent of the familiar inverse temperature. If only basic quantum mechanics were under consideration, the above would be only a theoretical exercise relating standard and quantum information entropies. But the Josephson junction equilibrium problem, as stated, is isomorphic to basic quantum theory, but is macroscopic in nature and allows for nonstandard information to be incorporated into the problem. Thus the state-level formulation becomes necessary.

5. SEPARABILITY AND TEMPERATURE

For a Josephson junction, the distribution on the allowed state space is akin to the canonical one, i.e., $P(\psi) d\psi \propto e^{-\gamma\langle H \rangle} d\psi$ for allowed states. The new "partition function" Q_R is determined by (14) with

$$\int d\psi = \int \prod_{i=1}^2 dc_i dc_i^* \delta(n_i - |c_i|^2) \quad (21)$$

to reflect the special constraint. This results in

$$Q_R \propto (n_1 n_2)^{1/2} e^{-\gamma(n_1 - n_2)\epsilon} I_0(2\gamma\kappa(n_1 n_2)^{1/2}) \quad (22)$$

The I_0 is a modified Bessel function. A clearer understanding results if it is noted that we have essentially obtained a distribution for the phase ϕ ,

$$p(\phi) d\phi \propto \exp[-2\gamma\kappa(n_1 n_2)^{1/2} \cos \phi] d\phi \quad (23)$$

Here it would be useful to make a connection to more physically accessible quantities. The coupling energy for a small junction and $n_1 = n_2$ is $E = J_0 \cos \phi$, with $J_0 = I_c/2e$ and I_c the critical current, for which approximate expressions exist.⁽¹⁰⁾ Thus, unspecified quantities such as κ can be replaced, using (2) or (3). So a more convenient form for (23) would be

$$p(\phi) d\phi \propto e^{-\gamma J_0 \cos \phi} d\phi \quad (24)$$

The above suggests that the junction is well described by a classical phase and the XY -type models, with partition function (5) and $p(\phi) d\phi \propto \exp(-\beta J_0 \cos \phi) d\phi$, so that in (24), γ would be playing the role of the inverse temperature. However, while γ and $\beta = 1/kT$ are both constants, they cannot be identified or said to be proportional. The $\cos \phi$ energy

expression does *not* stand for the eigenvalue of a Hamiltonian, but is a restricted state average of it. Thus there is a close similarity to a classical canonical distribution, but the parameter γ is nontrivially related to β . It can be seen that

$$\lim_{\beta \rightarrow \infty} \gamma = \infty, \quad \lim_{\beta \rightarrow 0} \gamma = 0 \quad (25)$$

as it must be, but the general expression for $\gamma(\beta)$ can be complicated.

The concept of “separability” is useful in understanding this difference, which reflects a distinction between classical and quantum states. Consider the description of a state in a combined system made up of A and B that have individual states S_A and S_B . The classical case is stated as “ A in S_A and B in S_B ,” and the description is separable into distinct subsystem descriptions. But for quantum states, superposition intervenes, giving $S_{AB} = \sum |S_A S_B\rangle$. This cannot be equated to a product of subsystem states; thus the quantum state-level description is nonseparable. Density matrix descriptions, in contrast, are separable, as $\rho_{AB} = \rho_A \otimes \rho_B$, since the combined basis states are products of subsystem basis states.

Consider thermal equilibrium between two systems that have negligible interactions, that are nonetheless present so as to make equilibrium possible. In such a case, the total energy expectation will be fixed, and probabilities will be a function of $H_A + H_B$:

$$\langle\langle H_A + H_B \rangle\rangle_{A+B} = \langle\langle H_A \rangle\rangle_{A+B} + \langle\langle H_B \rangle\rangle_{A+B} \quad (26)$$

where the $A+B$ subscript indicates that the expectation is in the combined system. For thermal equilibrium, we require that the individual subsystem expectations should not change if they become separated, or

$$\langle\langle H_{A,B} \rangle\rangle_{A+B} = \langle\langle H_{A,B} \rangle\rangle_{A,B} \quad (27)$$

With the separable descriptions, such as the density matrix, this leads immediately to equalization of temperatures,⁽¹¹⁾ $\beta_A = \beta_B$, from

$$e^{-\beta(E_A + E_B)} = e^{-\beta_A E_A} e^{-\beta_B E_B} \quad (28)$$

For the nonseparable case, $\beta \rightarrow \gamma$, and in general $\gamma_A \neq \gamma_B$. But the argument using (26) and (27) can still be followed.

One of the systems in consideration should be a collection of Josephson junctions, with phase probabilities described by a parameter γ . This will be taken to be in thermal equilibrium with a system that has a

known temperature dependence of its parameter γ' . The simplest choice would be a two-state system, where by applying (19) we get

$$\frac{1}{\gamma'} (1 - \gamma' E \coth \gamma' E) = -E \tanh \beta E \tag{29}$$

$\pm E$ being the energy eigenvalues. The total system “partition function” is

$$Q = \int d\psi \exp[-\bar{\gamma} \langle H \rangle - \bar{\gamma}' \langle H' \rangle] \tag{30}$$

We can now require that the component energy averages found through the total system distribution match the separate system expectations:

$$-\frac{\partial \log Q'}{\partial \gamma'} = -\frac{\partial \log Q}{\partial \bar{\gamma}'} \Big|_{\bar{\gamma} = \bar{\gamma}'} \tag{31}$$

establishes the connection to β . With

$$-\frac{\partial \log Q_R}{\partial \gamma} = -\frac{\partial \log Q}{\partial \bar{\gamma}} \Big|_{\bar{\gamma} = \bar{\gamma}} \tag{32}$$

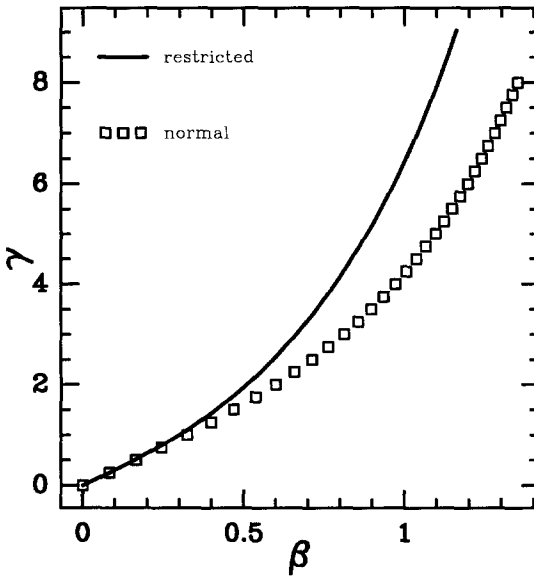


Fig. 1. $\gamma(\beta)$ for the component-magnitude constrained Josephson junction, which appears shifted compared to the analogous function for a two-state system with only an energy expectation constraint, Eq. (29). The parameter values were $n_1 = n_2 = 0.5$; $E = 1$, $\varepsilon = 0$, and $\kappa = 0.1$, in arbitrary energy units.

we finally have a relation between γ and β . Setting $\bar{\gamma} = \bar{\gamma}'$ in Q is necessary to have a true energy-expectation constraint. Figure 1 illustrates the result of such a calculation for the case of a single junction.

6. EXPERIMENTAL POSSIBILITIES

The main result of theoretical interest, addressing conceptual issues in fundamental statistical mechanics, is the demonstration that a state-level description, using the standard information entropy, is physically equivalent to the usual density matrix formulation for quantum statistical mechanics. It can be said that in this way the quantum entropy expression $\sigma = -\text{Tr } \rho \log \rho$ can be derived from the usual information entropy as a physically equivalent but separable, and thus much more convenient, expression. But the most general state-level treatment also opens up the possibility of handling unusual constraints in macroscopic approximations that give theories formally identical to basic quantum mechanics.

Weakly linked superconducting clusters, modeled as Josephson junction arrays, may provide such a case, necessitating a formulation of its statistical mechanics at the state-probability level, as the macroscopic quantum nature of the system provides us with usually unavailable information. This kind of question would be difficult to answer without use of maximum-entropy ideas, if at all possible. And it can be said that the above has confirmed that frustrated XY -type models⁽¹²⁾ can be approximately realized with junction arrays, with a more complicated temperature dependence than usually assumed.

The significance of the main physical result relating to Josephson junctions, the γ - β difference, can be explored further. This is a novel, purely statistical result, arising from the difference between classical and quantum states. As γ appears in (24), and any physical effect would be a result of this, it may seem that an effective "renormalization" of J_0 would account for it. But there are independent means of measuring this energy, beyond the fact that only statistical considerations enter into determination of $\gamma(\beta)$. Thus, certain equilibrium thermodynamic results, such as work on diamagnetic susceptibilities of granular systems,⁽⁴⁾ will have to be modified in the way that temperature dependence is exhibited.

It is difficult to say that a clear experimental confirmation of the quantum nature of states via the appearance of γ would be readily possible. The theoretical results on Josephson junction arrays tend to be qualitative when applied to real systems, and thus the relatively minor effects of unusual statistics could easily be washed out in comparing theories. To further complicate matters, in real systems, the single electron current or the junction capacitance cannot be easily ignored as we have done, since these will

change the behavior significantly.^(13,3) Therefore one is forced to conclude that the practical effect of the γ - β difference is negligible in most materials of interest, and an experiment would have to be specifically designed to confirm any such effect. Also, equilibrium properties are of comparatively less interest than the dynamics of junction arrays under current flow. In such cases, γ becomes quite irrelevant.

However, the complications presented by large collections of Josephson junctions could be mostly avoided if an idealized single junction were experimentally well approximated. With no external voltage source but finite temperature, the phase difference distributions in (23) and the classical form with β instead of γ can be compared. The dependence of γ on temperature would be similar to that in Fig. 1, and though the parameters for a real material may result in a less exaggerated difference from β , a nonnegligible effect can still be expected.

If the experimental difficulties are surmountable, the measurable quantity of interest would be the current at zero voltage, which is bounded by the junction critical current; $I = I_c \sin \phi$ with ϕ stationary.⁽¹⁰⁾ The distribution of this current value over many measurements would be predicted from (23).

One possibility would be to connect the two sides of the junction with the same superconducting material, forming a continuous superconducting loop except for the junction itself. The persistent current I in the absence of an external driving circuit would then not be subjected to resistive decay, and could be measured directly. Provided that the loop is large enough for phase coherence to disappear, the macroscopic phases on either side of the junction can be taken to vary independently. Alternately, if a nonsuperconducting connection between the junction elements is utilized, measurement of transient current behavior might be successfully related to the zero-voltage phase difference. The complexities in this particular approach arise from the necessity of current flow to establish a stationary phase difference, while resistive losses are also present.

A more extensive discussion of possible experimental details would be beyond the scope of this paper, but the conceptual basis for a test of the theory is in obtaining a good approximation for the zero-voltage current through a near-ideal Josephson junction. Such an experiment would also investigate the limits of applicability of the Feynman approximation (1) that is the basis for the discussion presented. A failure to confirm the expected phase distribution would be directly attributable to its breakdown. As this approach is a much encountered starting point for theoretical work about Josephson junctions, an experiment similar to that proposed could be of use in exploring aspects of Josephson junctions that have implications beyond statistical properties.

ACKNOWLEDGMENTS

I am grateful for a fellowship awarded by the Applied Physics Laboratory of the Johns Hopkins University. Thanks are due K. Moorjani and B. F. Kim for helpful discussions.

REFERENCES

1. J. Bohandy, B. F. Kim, F. J. Adrian, and K. Moorjani, *Phys. Rev. B* **39**:2733 (1989); F. J. Adrian, B. F. Kim, K. Moorjani, and J. Bohandy, *Phys. Rev. B* **43**:10328 (1991).
2. K. K. Mon and S. Teitel, *Phys. Rev. Lett.* **62**:673 (1989).
3. P. Hadley, M. R. Beasley, and K. Wiesenfeld, *Appl. Phys. Lett.* **52**:1619 (1988).
4. C. Ebner and D. Stroud, *Phys. Rev. B* **31**:165 (1985).
5. B. D. Josephson, *Phys. Lett.* **1**:251 (1962).
6. R. P. Feynman, *The Feynman Lectures on Physics*, Vol. III (1965), p. 21-14.
7. C. E. Shannon, *Bell Syst. Tech. J.* **27**:379, 623 (1948).
8. E. T. Jaynes, *Phys. Rev.* **106**:620 (1957); **108**:171 (1957).
9. E. T. Jaynes, *IEEE Trans. Syst. Sci. Cybernet.* **SSC4**:227 (1968).
10. T. Van Duzer and C. W. Turner, *Principles of Superconducting Devices and Circuits* (1981), p. 142.
11. R. Baierlein, *Atoms and Information Theory* (1971), p. 112.
12. S. Teitel and C. Jayaprakash, *Phys. Rev. B* **27**:598 (1983).
13. K. Wiesenfeld and P. Hadley, *Phys. Rev. Lett.* **62**:1335 (1989).